THE MATHEMATICAL
ASSOCIATION OF VICTORIA

MATHS TREATS

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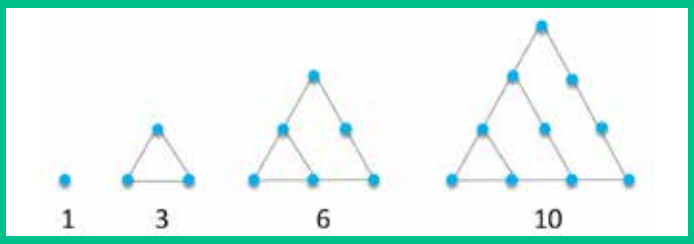
THE POSSUM

NUMBERS AS SHAPES

Figurate numbers are numbers made by arranging items into different geometric shapes. One of the simplest figurate numbers are square numbers. The term 'square number' refers to both the mathematical operation of multiplying a number by itself and the geometric arrangement. So the n^{th} square number has the value n^2 and that number of items can be laid out in the shape of a square. Exploring other, more complicated figurate numbers provides an entertaining way to explore various sequences and series.

TRIANGULAR NUMBERS

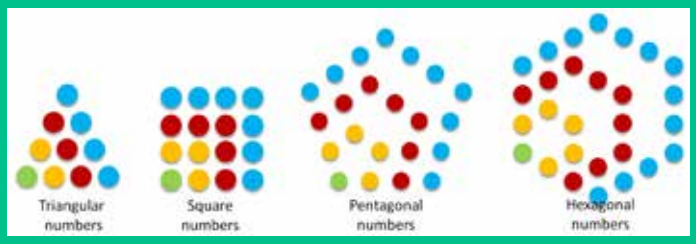


Ancient Greek mathematicians were interested in studying triangular numbers. Look up in the sky and you may see a flock of birds flying in triangular formation. Triangular numbers are formed by summing the terms in the arithmetic sequence 1, 2, 3, 4, … The sequence of triangular numbers is 1, 3, 6, 10, … Triangular numbers can also be formed geometrically, as depicted in the diagram above.

ACTIVITY

Calculate some more triangular numbers. Is there an end, or do the triangular numbers go on forever? Can you work out a formula to calculate the n^{th} triangular number? Show how a square number can be made from two consecutive triangle numbers. Can you find any other patterns with triangular numbers? Could you have three dimensional triangle numbers, and if you could, what would they look like? What about three dimensional square numbers?

OTHER FIGURATE NUMBERS



A polygon with five sides is called a pentagon, with six sides a hexagon, with seven sides a heptagon, and so it goes.

For example: The first three pentagonal numbers are 1, 5 and 12. You should be able to work this out from the illustration above. What are the first three hexagonal numbers?

ACTIVITY

Find some more pentagonal numbers. Can you find a formula or rule for the pentagonal numbers? If you are familiar with the sum of an arithmetic series, this may help.

What are some of the hexagonal numbers? Can you work out a rule for the sequence of hexagonal numbers? What about heptagonal numbers and other figurate numbers? Keep going until you can see a pattern which might help you come up with a general formula for figurate numbers. Can you find other ways to arrange items (or dots) to make, say, hexagonal, octagonal or other polygonal numbers? What other kinds of geometric numbers could you invent?

REFERENCES AND FURTHER READING

TRIANGULAR NUMBERS

Triangular numbers, https://en.wikipedia.org/wiki/Triangular_number

Fascinating facts about triangular numbers, www.shyamsundergupta.com/triangle.htm

Investigating triangular numbers with technology, http://jwilson.coe.uga.edu/EMAT6680Su11/Kitchings/CK6690/Triangular%20numbers%20formula.html

FIGURATE NUMBERS

Figurate number, https://en.wikipedia.org/wiki/Figurate_number

2D and 3D figurate numbers, http://oeis.org/wiki/Figurate_numbers

Investigating figurate numbers with technology, http://jwilson.coe.uga.edu/EMAT6680Su11/Kitchings/CK6690/Figurate%20Numbers.html

IMAGES

Leadbeater possum - Steve Kuiter
Other images - Pixabay